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Applying to (8) Lagrange's equations of typical form

$$\frac{d}{dt}\frac{dT}{d\dot{\varphi}_n}-\frac{dT}{d\varphi_n}=\frac{dV}{d\varphi_n},\qquad n=1,2,3, \tag{9}$$

the φ_1 function is

$$\begin{aligned}\{m_1(a_1^2+k_1^2)+4a_1^2m_2+4a_1^2m_3\}\ddot{\varphi}_1&+2a_1a_2m_2\cos(\varphi_1-\varphi_2)\ddot{\varphi}_2+2a_1a_2m_2\sin(\varphi_1-\varphi_2)\dot{\varphi}_2^2\\ &+4a_1a_2m_3\cos(\varphi_1-\varphi_2)\ddot{\varphi}_2+4a_1a_2m_3\sin(\varphi_1-\varphi_2)\dot{\varphi}_2^2+2a_1a_3m_3\cos(\varphi_1-\varphi_3)\ddot{\varphi}_3\\ &+2a_1a_3m_3\sin(\varphi_1-\varphi_3)\dot{\varphi}_3^2=-ga_1(m_1+2m_2+2m_3)\sin\varphi_1,\end{aligned}\tag{10}$$

and the φ_2, φ_3 functions are

$$\begin{aligned}\{m_2(a_2^2+k_2^2)+4a_2^2m_3\}\ddot{\varphi}_2&+\{2a_1a_2m_2\cos(\varphi_1-\varphi_2)+4a_1a_2m_3\cos(\varphi_1-\varphi_2)\}\ddot{\varphi}_1\\ &-2a_1a_2m_2\sin(\varphi_1-\varphi_2)\cdot\dot{\varphi}_1^2-4a_1a_2m_3\sin(\varphi_1-\varphi_2)\cdot\dot{\varphi}_1^2+2a_2a_3m_3\cos(\varphi_2-\varphi_3)\ddot{\varphi}_3\\ &+2a_2a_3m_3\sin(\varphi_2-\varphi_3)\dot{\varphi}_3^2=-ga_2(m_2+2m_3)\sin\varphi_2,\end{aligned}\tag{11}$$

$$\begin{aligned}m_3\{(a_3^2+k_3^2)\ddot{\varphi}_3&+2a_2a_3\cos(\varphi_2-\varphi_3)\ddot{\varphi}_2-2a_2a_3\sin(\varphi_2-\varphi_3)\dot{\varphi}_2^2+2a_1a_3\cos(\varphi_1-\varphi_3)\ddot{\varphi}_1\\ &+2a_1a_3\sin(\varphi_3-\varphi_1)\cdot\dot{\varphi}_1^2\}=-m_3ga_3\sin\varphi_3.\end{aligned}\tag{12}$$

Let φ_1, φ_2, φ_3 be so small that one may employ the approximations $\sin\varphi_1=\varphi_1$, etc.; $\cos\varphi_1=1$, etc.; $\dot{\varphi}_1^2=0$, etc.; $\dot{\varphi}_1\dot{\varphi}_2=0$, etc. Then (10), (11), (12) may be written

$$\begin{aligned}\{m_1(a_1^2+k_1^2)+4a_1^2(m_2+m_3)\}\ddot{\varphi}_1+2a_1a_2m_2\ddot{\varphi}_2+4a_1a_2m_3\ddot{\varphi}_2+2a_1a_3m_3\cos(\varphi_1-\varphi_3)\ddot{\varphi}_3&\\ =-ga_1(m_1+2m_2+2m_3)\varphi_1,&\end{aligned}\tag{13}$$

$$\{2a_1a_2m_2+4a_1a_2m_3\}\ddot{\varphi}_1+\{m_2(a_2^2+k_2^2)+4a_2^2m_3\}\ddot{\varphi}_2+2a_2a_3m_3\ddot{\varphi}_3=-ga_2(m_1+2m_3)\varphi_2,\tag{14}$$

$$m_3\{(a_3^2+k_3^2)\ddot{\varphi}_3+2a_2a_3\ddot{\varphi}_2+2a_1a_3\ddot{\varphi}_1\}=-ga_3m_3\varphi_3.\tag{15}$$

These are the three equations of small motion in the general form of the problem.

In the present problem, $m_2=0$; $k_1^2=a_1^2/3$, $k_2=0$, $k_3^2=\frac{2}{5}a_3^2$; and we may put $m_1=m_3=m$; also $2a_1=2a$, $2a_2=b$, $2a_3=2c$, and (13), (14), (15) are

$$\begin{aligned}\tfrac{16}{3}a\ddot{\varphi}_1+3g\varphi_1+2b\ddot{\varphi}_2+2c\ddot{\varphi}_3&=0,\\ 2a\ddot{\varphi}_1+b\ddot{\varphi}_2+g\varphi_2+c\ddot{\varphi}_3&=0,\\ 2a\ddot{\varphi}_1+b\ddot{\varphi}_2+\tfrac{7}{5}c\ddot{\varphi}_3+g\varphi_3&=0;\end{aligned}$$

or, using another notation of differential equations,

$$\begin{aligned}(\tfrac{16}{3}aD^2+3g)\varphi_1+2bD^2\varphi_2+2cD^2\varphi_3&=0,\\ 2aD^2\varphi_1+(bD^2+g)\varphi_2+cD^2\varphi_3&=0,\\ 2aD^2\varphi_1+bD^2\varphi_2+(\tfrac{7}{5}cD^2+g)\varphi_3&=0.\end{aligned}$$

Eliminate φ_1 and φ_2; then,

$$\{8abcD^6+g(20ab+18bc+52ac)D^4+g^2(80a+45b+63c)D^2-45g^3\}\varphi_3=0.\tag{16}$$

Let

$$\varphi_3=L_p\cos(\lambda t+\alpha_p),$$

and after substitution in (22) and simplifying, put $\lambda^2=\mu$, and we have

$$8abc\mu^3-g(20ab+18bc+52ac)\mu^2+g^2(80a+45b+63c)\mu-45g^3=0,$$

the cubic equation required.

Also solved by J. B. REYNOLDS and W. E. CEDERBERG.

259 (Number Theory). Proposed by E. E. WHITFORD, College of the City of New York.

If p is relatively prime to 10, and if any multiple of p consisting of n digits has its digits permuted cyclically, the number thus formed is also a multiple of p; the number n to be determined by the congruence $10^n\equiv 1 \pmod{p}$. For example, 481, 814, and 148 are each multiples of 37.

Solution by Benjamin F. Yanney, College of Wooster, Wooster, Ohio.

Let the digits in the initial order be

$$a_n,\ a_{n-1},\ \cdots,\ a_1.$$

Then by hypothesis,

$$a_n10^{n-1} + a_{n-1}10^{n-2} + \cdots + a_210 + a_1 \equiv 0 \pmod{p}.$$

Multiply each member of the congruence by 10, remembering that $10^n \equiv 1 \pmod{p}$, and place the digit a_n in units place. We thus secure one cyclic permutation. Another cyclic permutation is secured by multiplying again by 10; and so on. This completes the proof.

It will be observed that the theorem may be generalized by multiplying all the terms to the left of any specified term by $10^{\kappa n}$, in the case of any cyclic permutation, where κ is any positive integer. Thus, in the example given, 480001, 800000014, and 140000000008 are also each multiples of 37. By successive application of this method, we may obtain different types of cyclic permutations. Thus, 400000080001 is a multiple of 37. We may have other than cyclic permutations, with ciphers, by multiplying any one or more terms of the above congruence by $10^{\kappa n}$, where κ can have a different value for each term multiplied. Thus, 80401 is also a multiple of 37. It is interesting to note in this more general application that no two integers of the original number can ever collide.

Also solved by L. C. Mathewson, Philip Franklin, W. R. Ransom, Frank Irwin, Paul Capron, Horace Olson, and C. C. Yen.

262 (Number Theory). Proposed by C. N. SCHMALL, New York City.

If x, y, z, are 3 integers, consecutive among the integers prime to 3, show that

$$x(x - 2y) - z(z - 2y) = \pm 3.$$

Solution by Edward H. Vance, Graduate Student, Ithaca, N. Y.

Let $v - 1$ be any number divisible by 3, then any set of three integers consecutive among the numbers prime to 3 may be represented by one of the following sets:

$$v - 2,\ v,\ v + 1;\quad v,\ v + 1,\ v + 3.$$

Substituting $v - 2$, v, $v + 1$ for x, y, z, respectively, in the lefthand side of the given equation we have

$$x(x - 2y) - z(z - 2y) = 3.$$

Substituting v, $v + 1$, $v + 3$ for x, y, z, respectively, we have

$$x(x - 2y) - z(z - 2y) = -3.$$

Also solved by Paul Capron, N. P. Pandya, Louis O'Shaughnessy, Lewis Clark, E. F. Canaday, George W. Hartwell, J. L. Riley, Albert G. Rau, Herbert N. Carleton, Horace Olson, and V. M. Spunar.

QUESTIONS AND DISCUSSIONS.

Send all communications to U. G. Mitchell, University of Kansas.

DISCUSSIONS.

I. On Making Mathematical Results More Available for Engineers.

By Willis Whited, Harrisburg, Pennsylvania.

Some time ago I received a circular from the Mathematical Association of America regarding the *Annals of Mathematics*. I like very much the idea of a